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THREE LECTURES

ON

RATIONAL MECHANICS;

OR THE

THEORY OF MOTION.

BY

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TO

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THE

FOLLOWING LECTURES

ARE

AFFECTIONATELY INSCRIBED,

BY THEIR FRIEND,

THE AUTHOR.

LECTURE I.

THE LAW OF INERTIA.

IT is one of the sayings of Aristotle, that "If motion be not understood, we cannot but remain ignorant of nature." Though Aristotle may not, it is true, have seen all the depth and beauty of his own remark; yet has its justness been illustrated and confirmed by the whole subsequent history of natural philosophy. Indeed, it has now become a maxim, that "the true theory of motion is the key to nature." It was with this key, first constructed and fitted for use by Galileo, that Newton, the great high-priest of nature, entered into her most profound recesses, and laid open her most sublime secrets to the admiration of mankind.

Just in proportion, then, as our notions of the great laws of motion are clear, precise, and steady, will nature become transparent to our view, and unfold its beauty in cloudless splendor. The truth of this remark is seen in the consequences of every improvement in the science, or theory of motion. For every such improvement, like an improvement in the telescope, has extended the range, and cleared up the field, of our vision into the wonderful mechanism of the material universe. We shall endeavor, in the present lecture, to illustrate and establish this position, in regard to the great fundamental law of *inertia;* which is usually, and very pro-

perly, laid down by philosophers, as the first law of motion.

What, then, is meant by the *inertia* of matter, and the law which springs therefrom? Look around you, young gentlemen, and you will at once be struck with the vast difference between two classes of objects. You will see organized, living, spiritual substances; and substances which are wholly dead and passive. In the one you will perceive, (*i. e.* by its manifestations,) an inherent principle of activity; in the other, you will discover only a lifeless mass of matter. This we call inert; because it is wholly destitute of a power to act. An instrument of power it may be, and indeed often is; but in and of itself it is perfectly powerless. Hence, if placed at rest in any point of space, it will continue in a state of rest forever, unless driven from its position by some extraneous force. This constitutes the first branch of the great law of *inertia.* This branch of it is easily grasped and retained by the mind. So far, then, we have no difficulty—we are in danger of no error, of no illusion.

But the second part of this great law, is not so easily conceived, nor so readily held with a firm grasp. It is to be reached and realized only by an act of sustained reflection; and even after it has been thus reached and realized, it will often prove an exceedingly unstable element in our minds, and be often found to desert us when our logic most stands in need of its presence, and its power. As we are about to enter upon this slippery ground, then, let us determine to do so with a clear eye, and a firm step. Let us fix our minds upon this second branch of the law of *inertia;* and hold them to it, until it becomes as clear and fixed as the noonday sun. Let us look at it, until it ceases to seem a vague, dim, shadowy, wavering element of thought, and comes to stand

before us, clear and bright, like the adamantine pillars of heaven and earth. Then shall we be able to see our way clearly, where, otherwise, clouds and darkness would overspread our path. Then shall we be able to advance steadily, and with conscious power, where otherwise we should be compelled to grope our way timidly amid the manifold mists and obscurities of nature.

The idea we should thus realize is this: If a body be put in motion, it will continue to move in a right line forever, and with a uniform velocity; unless checked in its career, or turned from its direction, by some extraneous force. This is the second part of the great law of *inertia.* It is one thing to hear it—it is one thing to think we understand it—and quite another to possess that full, clear, and perfect comprehension of it, which can only be attained to by severe meditation. I do not wish to deal in paradox, or to startle you by extravagant assertions of any kind; but yet I must say it, because it is the profound conviction of my mind, that, although this law is in everybody's mouth, not one natural philosopher in a hundred is in full possession of its import; and even those who have realized it the most fully, are often betrayed into errors of logic, and false interpretations of the phenomena of nature, by permitting it to elude their attention. The truth of this remark will appear, I trust, even before the conclusion of the present lecture.

The ancients do not seem to have caught the most distant glimpse of the great law in question. Every portion of the physics of Aristotle betrays a profound ignorance of its existence. His well-known division of motion into *violent* and *natural,* is one instance among many, of his having wandered in the dark, from a want of the light of this law. So far from perceiving that all motion is natural to body, as perfectly natural, in-

deed, as is a state of rest; he supposed that some sort of violence was often done to matter by forcing it to move, and that such motion would run down of itself, without any cause of its extinction. It would languish and gradually die away, he imagined; because its existence was contrary to the nature of body. Hence, instead of attempting to explain the phenomena offered to the senses by any *known* principle, or even according to any clearly defined idea, he had recourse to the fanciful distinction between *natural* and *violent* motion; one of the most vague, dim, and impalpable notions that ever cast its shadow over the brain of man.

The later writers of the Aristotelian school of physics expressed this famous distinction in verse. The law of natural motion was:—

> "Cool at the first, it warm and warmer glows."

That of violent motion:—

> "Hot at the first, then barely warm, then cold."

The poetry may not be good; it is certainly better than the philosophy. If we had been told that some motion runs down of itself, while other motion is accelerated, because all this is natural, we should have received all the information which could possibly be derived from the distinction in question; and that is precisely no information at all. And then, this pompous nothing is set forth in such a distressing circumlocution of words; it had been far better expressed thus: *it is so, because it is so.* For this would have shown the idea as it is in itself, or rather as it is in its own naked nothingness; and relieved the intellectual void from the pressure of mere words, that it might return to its old craving after ideas.

If Aristotle had possessed the law of *inertia*, it would

have spared him the labor of constructing a mass of false logic, which it would require a volume to point out. One or two instances only, shall I at present adduce. "Circular motion," says he, "is the only one which can be endless." Now, why is this the case? Why is it impossible, in the very nature of things, for motion to proceed in a right line forever? "Because," says Aristotle, "that cannot be doing which cannot be done; and therefore it cannot be that a body should be moving towards a point, (*i. e.* the end of an infinite straight line,) whither no motion is sufficient to bring it." Thus the law of *inertia*, which is really true, is shown to be an inherent impossibility. Is it at all wonderful, that men should have failed to institute a close and searching examination into nature, when they could so easily mistake fancies for laws, and the great problems of the material world could be so easily settled by a few admirable strokes of logic?

It was necessary to come to the conclusion that "circular motion may be endless," in order to account for, or rather not to contradict, the apparent motions of the heavenly bodies. These were supposed to be natural in the highest sense of the word; and, therefore, not liable to decay, like the motions of earthly objects. They were supposed, indeed, to constitute a distinct class of motions, and to be governed by wholly different laws, from the motions of the sublunary sphere. It is even argued by Aristotle, that the motions of the heavenly bodies must be uniform, because it is beneath the dignity of such divine objects to move sometimes faster and sometimes slower. Thus it was proved, that their motion was not only circular, but also eternal; for so long as the motion is uniform it can never come to an end.

So far from perceiving, that if the motion of a body be not disturbed, it will move in a right line forever,

Aristotle has attempted to demonstrate that it is utterly impossible for it to move in a vacuum at all. To give the substance of his argument: The rarer a medium is, the faster a body will move therein; but since a perfect vacuum is infinitely rarer than any medium in which motion has been observed, the motion of a body in it would be infinitely more rapid than in a resisting medium; that is to say, it would pass from one point to another *instantly*, which is impossible. Thus he argues, that if a body could move in a vacuum at all, it must move with an infinite velocity, or pass over some space in no time, which is impossible; and hence, he concluded, it cannot move in a vacuum at all.

You have, no doubt, heard the famous exclamation of the affected lover in one of Dryden's plays:

> "My wound is great because it is so small:"

And also the extemporary line of the Duke of Buckingham, by which this nonsense was wittily exposed:

> "It would be greater were it none at all."

Now the argument of Aristotle is precisely the logical antipode of this; for he proves that in a vacuum the motion of a body would be so great, it could not be at all.

The principle of Aristotle on which the foregoing argument was based, namely, that the velocity of body is inversely proportioned to the medium through which it moves, long maintained its hold on the scientific world. Even as late as the sixteenth century, Cardan was so well persuaded of its truth, that he proposed to determine the relative densities of air and water, by observing the respective velocities with which a body would fall through them. He supposed, for example, that if a body would fall ten times as fast in air as in water; then water is ten times as dense as air. Galileo

proposed to try the experiment with a cork; and this put an end to the theory.

Aristotle was sadly perplexed to account for the continued motion of a body, after it had escaped from the moving force. Adopting the maxim, that nothing can act where it is not, he imagined that actual contact is necessary, not only to the production, but also to the propagation, of motion. Hence he could not conceive, why a stone should continue to move after it had departed from the hand by which it was thrown; or why any body should continue its motion longer than it remained in contact with the instrument by which it was moved. This difficulty, which so plainly arose from an ignorance of the law of *inertia*, led him to the strange conclusion, that falling bodies are driven downward by the air rushing in behind and urging them on. The same feeble conceit, he applied to the continued motion of all bodies, which pass through a resisting medium. He supposed that the original impulse or force produced their motion; and that it was kept up by the resisting medium rushing in behind the bodies, and impelling them forward against itself!

If now we pass from the physical speculations of Aristotle down to the age of Kepler, we shall find that the same error respecting the first law of motion, spread a darkness, deep and dense, over one of the otherwise most brilliant epochs in the history of science. During this period, also, it was universally supposed that, as force is necessary to move a body, so a perpetual supply of force is required to keep it in motion. The whole of Kepler's reasoning is based on this assumption, on this virtual denial of the law of *inertia*. The idea that each planet, by virtue of an original impulse, and the law of *inertia*, tended to move in a right line eternally, and with a uniform velocity, constitutes no part of the theory of

this splendid astronomer. Hence he had recourse to a certain imaginary quality called *force*, which, he supposed, attended each planet in its course, and impelled it in the direction of its motion. According to this view, if these forces were annihilated, the planets would soon cease to move, and the clock-work of the solar system run down. This error alone, which assumes that force is necessary to the preservation, as well as to the production of motion, was sufficient to have rendered the efforts of even a thousand Keplers unavailing to solve the stupendous problem of the material universe. So long as this error prevailed in his mind, his genius, lofty and soaring and sublime as it was, must have been baffled and driven back by the insuperable difficulties of the subject. The more ardent his aspirations, indeed, the more strenuous his exertions, the more gigantic his efforts, the more severely must he have felt the manacles and chains that bound him to the earth. And it was only after its deliverance from this error, that the human mind, by the grandest flight it had ever taken, reached the summit of the world; and thence, far above the fogs and mists of the lower atmosphere, surveyed the unutterable glory of the creation.

So far from supposing that the earth, whether resting or moving, is an inert mass of matter; this wonderful man imagined that it is really animated—a huge animal—a prodigious monster—swelling and puffing and blowing, as it finds its way through the infinite depths of space. This was no metaphor with him, but a real belief. "If any one," says he, "who has climbed the peaks of the highest mountains, throw a stone down the very deep clefts, a sound is heard from them; or if he throw it into one of the mountain lakes, which, beyond doubt, are bottomless, a storm will immediately arise, just as when you thrust a straw into the nose or ear of a ticklish

animal, it shakes its head or runs shuddering away. What so like breathing, especially of those fish who draw water into their mouths, and spout it out again, through their gills, as that wonderful tide. For although it is so regulated, according to the course of the moon, that in the preface to my commentaries on Mars, I have mentioned it as probable that the waters are attracted by the moon, as iron is by the loadstone; yet if any one uphold that the earth regulate its breathing according to the motion of the sun and moon, as animals have daily and nightly alternations of sleeping and waking, I shall not think his philosophy unworthy of being listened to; especially if any flexible parts should be discovered in the depths of the earth to supply the functions of lungs or gills." The imagination of Kepler, by which the earth itself, and all things therein, were animated, was utterly irreconcilable with the idea that matter is wholly *inert*, and therefore equally indifferent to motion or to rest.

The profoundly meditative mind of Galileo, was the first to open on the grand idea of *inertia*, and cause its light to dawn on the world. This sublime conception, however, was slowly formed in the mind of Galileo, and only after many false notions and blunders had preceded it in his imagination. "Returning to the subject of motion," says he, "I was utterly without a fixed principle, from which to deduce the phenomena I had observed; I have hit upon a proposition which seems natural and likely enough." The principle he thus hit upon is this: that the velocity of the falling body increases in the proportion of its distance from the point whence it began to fall. Thus, if at the distance of ten feet, the velocity be as one, then at the distance of twenty feet, it would be as two; and so on, the velocity always being as the distance of the body from the point

of departure. A knowledge of the law of *inertia* would have enabled him not only to correct this error, but also to set out from a clear and fixed principle in the determination of the other laws of motion, and the explanation of the real phenomena of nature. It would have revealed to him the true relation between the velocity acquired, and the spaces described, by a falling body. He would have seen that if a certain number of feet as a were described in one unit of time, by a falling body, it would pass over $2\ a$ in the next unit of time by virtue of the law of inertia alone, while another space equal to a would be described in consequence of the continued action of gravity. But the velocity generated in two units of time, by the same force, would be only double of that produced in one unit of time. Thus while the velocity would be double, the space would be quadruple; that is, the space would be to the velocity as 4 to 2; it would be, not as the velocity simply, but as the square of the velocity.

It is not known at what time Galileo saw through this error. We only know that in his dialogues on motion, which contain the true theory, this false notion is put into the mouth of Sagredo, on which Salviati remarks: "Your discourse has so much likelihood in it, that our author himself did not deny to me when I proposed it to him, that he also had been some time in the same mistake. But that which I afterward extremely wondered at was, to see discovered in four plain words, not only the falsity, but the impossibility of a supposition carrying with it so much of seeming truth, that, although I proposed it to many, I never met with any one but did freely admit it to be so; and yet it is as false and impossible, as that motion is made in an instant; for if the velocities are as the spaces passed, these spaces will be passed in equal times, and consequently

all motion must be instantaneous." It was thus, and not by the aid of the law of *inertia*, that Galileo was delivered from his error, or, at least, that he labored to disprove it.

The "four plain words," however, in which Galileo sets forth his conclusion, are not so clear to ordinary apprehension. And his biographer, who attempts to make these four plain words plainer, involves them, if possible, in still greater obscurity. "The following manner of putting this reasoning," says he, "will perhaps make the conclusion clearer. The velocity at any point is measured by the space that would be passed in the next moment of time, if the motion be supposed to continue the same as at that point. At the beginning of time, when the body is at rest, the motion is none; and, therefore, on this theory, the space passed in the next moment is none; and thus it will be seen, that the body cannot begin to move according to the supposed law." I will only say, in passing, that this attempt to disprove the possibility of motion from the supposition of the law in question, is directed against the truth. For it is well known, that "the velocity at any point" is actually measured in precisely the manner specified by the learned author; that is, "by the space passed in the next moment of time, if the motion be supposed to continue the same as at that point." Hence, if his argument prove anything, it clearly deduces, from the true law itself, the impossibility of motion in regard to falling bodies.

This reminds one of the world-famous sophism, by which one of the ancients endeavored to disprove the possibility of any motion. If a body move, says he, it must either move in the place where it is, or in the place where it is not. It cannot move in the place where it is not, because it is not there to move; and it

cannot move in the place where it is, because while in the place where it is, it stands still. Hence it cannot move at all. Sir William Hamilton says, that this argument has never been answered; and seems to entertain the opinion, that it is one of those puzzles which must forever prove too hard for the human mind. For my part, it seems to me, that if this sophism has never been answered, it is because everybody feels that such things are made to be laughed at, and not refuted. But if, out of respect to the great fame of Sir William Hamilton, we should be disposed to refute it, we might say: True, a body can neither move where it is, nor where it is not; but, then, can it not move from the place where it is, *to* the place where it is not? The sophist was careful to exclude this, *the only real question*, from the terms of his cunningly devised alternative; and thereby made a pitiful show of argument, which is sufficiently refuted by the simple fact of motion.

But to return from this short digression. Galileo first put forth the true doctrine on the subject of *inertia* in 1638, after his mind had been turned from the study of astronomy, by the persecutions of the Church of Rome. "Conceive a movable body," says he, "upon a horizontal plane, and suppose all obstacles to motion to be removed; it is then manifest, from what has been said more at large in another place, that the body's motion will be uniform and perpetual upon the plane, if the plane be indefinitely extended." This was the first announcement ever made of the great law of *inertia*.

Little could Galileo have seen, however, at that early day, of the wonders it was destined to work in the whole mass of mechanical knowledge. Little did he imagine that the law thus unostentatiously propounded by him was destined to become a constituent element in the true theory of the universe itself. Little did he imagine

that it was destined to become an instrument in the hand of a Newton to strike a light from nature, which should utterly overwhelm and confound the persecuting bigots who had driven him from the study of astronomy. Little did he imagine, that by these seemingly humble inquiries of his, he had done more to advance the great cause in which he had so gloriously labored, and so ingloriously suffered, than if his immortal powers had been directly exerted upon the mechanism of the heavens. And little did his persecutors imagine, that the feeble old man, bending beneath the weight of years, retired before the storm of their impotent fury, only to prepare for them a still more signal defeat and overthrow. But so it was; and so it ever will be. The powers of darkness may rage; the powers of light will still prevail. Though persecuted, down-trodden, and apparently crushed, the unconquerable spirit of man will still *think;* and, *thinking*, claim to hold communion with the glory of God's works. And that glory once seen, and once felt, will have a voice and an utterance, which all the thunders of the Vatican cannot silence.

We have now taken a rapid view of the great law of *inertia*, in its relation to the mechanical philosophy of the ancients, as expounded and maintained by Aristotle; which, with all its inherent infirmities, its manifold contradictions and absurdities, prevailed for so many centuries over the human mind. We have also considered it in relation to the epoch, which has been rendered forever illustrious by the labors of a Tycho Brahe, a Kepler, and a Galileo. We shall hereafter witness its still more glorious developments, when we pass from the study of mechanical philosophy to that of physical astronomy. In the mean time, let us endeavor to realize the full import of this great idea, that it may be a lamp to our feet through all the dark places of philosophy.

In order to do this with success, we must learn to distinguish between the nature of *action* and *motion*, of *force* and *velocity*. These are *the* two elements with which mechanical philosophy has to do; and as they are essentially different in nature and kind, so we must habitually distinguish them in our thoughts, if we would pass through the fair regions of nature with a clear vision and a deep insight. Here they are, then; look at them, and see them. The one is an exertion of power; the other exists in that which is wholly destitute of power. Force is an exercise of power; velocity is the effect of force. Power acts, it does not move; body moves, it does not act. Force is necessary to account for the rise of velocity or motion; the absence of all force accounts for the continuance of motion. Body passively yields to forces, and moves from place to place. Its motion, therefore, is not a *doing;* it is a *suffering*. It merely *suffers* a change of place when it moves. And it is then as perfectly passive when it is in motion, as when it is at rest; and when we speak of "the *action* of body," this should be understood as a metaphor, or figure of speech. Otherwise, we shall mistake rhetoric for logic, and become entangled in mere words.

This may seem like a metaphysical nicety, young gentlemen, which has little to do with mechanical philosophy; but we shall see, in the next lecture, that it is a nicety which cannot be overlooked with impunity. We shall see, that by an inattention to this distinction, the greatest luminaries of science have been eclipsed in regard to the measure of "the force of moving bodies." We shall see, that after having labored for more than fifty years to determine such a measure, they finally settled down upon a wrong one, in which the scientific world has now acquiesced for more than a century. We shall see, that the light of this distinction between the

nature of *force* and *motion*, (or rather of the great law of *inertia*, of which this light is but one of the rays,) would have conducted them, with perfect clearness, through all the phenomena by which they were so sadly perplexed. They might have given to the theory of motion, even in regard to "the force of moving bodies," as it was called, all the precision, clearness, and beauty of a demonstrative science. They might have brought it out from the cloudy region of a false metaphysics; and presented it to the mind as a radiant and clearly defined body of truth, without either shade or shadow.

LECTURE II.

THE FORCE OF MOVING BODIES.

"DESCARTES had estimated the force of a moving body," says Dr. Playfair, "by the quantity of its motion, or by the product of its velocity into its mass. The mathematicians and philosophers who followed him did the same, and the product of these quantities was the measure of force universally adopted. No one, indeed, had ever thought of questioning the conformity of this measure to the phenomena of nature, when, in 1686, Leibnitz announced, in the *Leipsic Journal, the demonstration of a great error committed by Descartes and others, in estimating the force of moving bodies.* In this paper, the author endeavored to show, that the force of a moving body is not proportional to its velocity simply, but to the square of its velocity; and he supported this new doctrine by very plausible reasoning."*

Indeed, the reasoning of Leibnitz was so exceedingly plausible, that it seemed to shake the old theory to its foundation, and gave rise to a controversy which vexed the scientific world for more than half a century. "The mathematical world," says the same elegant writer, "could not look with indifference on a question which seemed to affect the vitals of mechanical science, and soon separated into two parties, in the arrangement of

* Progress of Mathematical and Physical Science, p. 52.

which, however, the effects of national predilection might easily be discovered. Germany, Holland, and Italy declared for the *vis viva;* England stood firm for the old doctrine; and France was divided between the two opinions. No controversy, perhaps, was ever carried on by more illustrious disputants; Maclaurin, Stirling, Desaguliers, Jurin, Clarke, Mairan, were all engaged on the one side, and on the opposite were Bernoulli, Herman, Poleni, Gravesande, Muschenbroek."

The reasoning of Leibnitz, which is regarded by D'Alembert as sound to a certain extent, is well stated by Playfair in the following passage: "A body," he says, "projected upward against gravity, with a double velocity, ascends to four times the height; with a triple velocity, to nine times the height, and so on; the height ascended to being always as the square of the velocity. But the height ascended to is the effect, and is the natural measure of the force, therefore the force of a moving body is as the square of its velocity. Such was the first reasoning of Leibnitz on this subject—simple and apparently conclusive; nor should it be forgotten that, during the long period to which the dispute was lengthened out, and notwithstanding the various shapes which it assumed, the reasonings on his side were nothing more than this original argument, changed in its form, or rendered more complex by the combination of new circumstances, so as to be more bewildering to the imagination, and more difficult either to apprehend or to refute."

The controversy raged with great warmth, and, considering the nature of the subject, with no little rancor. In 1743, it was supposed to have been finally settled. "It was now," says Dr. Playfair, referring to this period, "drawing to a conclusion, and in effect may be said to have been terminated by the publication of D'Alembert's

Dynamique in 1743." Dugald Stewart also, in his Account of the Life and Writings of Dr. Reid, has assigned to D'Alembert the honor of having brought this celebrated controversy to a conclusion, by means of a solution in which the scientific world has acquiesced. Such is, indeed, the general opinion; and we are told that the doctrine of Leibnitz, "under certain conditions, is to be considered as a matter completely established." The conclusions supposed to be so firmly established by D'Alembert, have even found their way into our elementary works on mechanical philosophy. It may seem presumptuous, then, for any one to agitate this point again, or to call in question the celebrated solution of D'Alembert. I certainly should not do so, if I did not believe that I have found the means of clearly demonstrating the fallacy of that solution, and also of giving the true solution in its stead.

In the opinion of D'Alembert, and those who have followed him, the reasoning of Leibnitz merely shows that the "square of the velocity," is one of the measures of "the force of a moving body." His view of this subject may be more fully presented in the words of Playfair. If the force of a moving body "be employed in putting in motion another body, which is itself free to move, no doubt remains that the force is as the velocity multiplied into the quantity of matter. So also, if the force of the moving body is to be opposed by a resistance like that of gravity, the length of time that the motion may continue is one of its measurable effects, and that effect is, like the former, proportional to the velocity. There is a third effect to be considered, and one which always occurs in such an experiment as the last—the height to which the moving body will ascend. This limitation gives to the force a definite character, and it is now measured by the square of the velocity."

From these facts and statements, the author concludes that we have "two results, or two values of the same thing, the force of a moving body, which are quite different from one another." Such is the conclusion to which the master mind of D'Alembert has conducted, not only Dr. Playfair, but the great majority of scientific men.

In opposing the solution of D'Alembert, now so generally received, I shall endeavor to show with the utmost mathematical clearness and precision, that the doctrine of Leibnitz is not true under any circumstances, or within any limits; that it has merely the shadow, and not the substance, of a foundation in the nature of things. I shall endeavor to restore the old theory of motion to its original simplicity and beauty, as well as to invest it with a new lustre, by causing it to explain the very phenomena which have been supposed to limit and control its universality. This may be most effectually accomplished, first, by exposing the errors on which the commonly received solution is founded; and, secondly, by *demonstrating* the solution to be substituted in its place.

D'Alembert sets out with the declaration, that the dispute in question has been merely about words; and since his time this declaration has been frequently repeated. Voltaire, in a Memoire presented to the Academy of Science in 1741, was the first to set the fashion of asserting that the dispute under consideration was exclusively about words. He has since been followed, in this view of the subject, by those who were vastly superior to himself in mathematics and philosophy. Even D'Alembert has adopted the conclusion of the wit and the poet; and by incorporating it into his celebrated *Dynamique,* has rendered it a part and parcel of the commonly received doctrine.

Those who maintain this opinion, however, have not been able to do so without falling into the most manifest confusion and self-contradiction. Playfair (to select a conspicuous example from among many,) may be mentioned as a striking illustration of the truth of this remark. "It was therefore evident," says he, "that however much their language and words were opposed, *their ideas or opinions exactly agreed.* In reality, the two parties were not at issue on the question; their positions, though seemingly opposite, were not contrary to one another; and after debating for nearly thirty years, they found out this to be the truth. That the first men in the scientific world should have disputed so long with one another, without discovering that their opposition was only in words, and that this should have happened, not in any of the obscure and tortuous tracts through which the human mind must grope its way in anxiety and doubt, but in one of the clearest and straightest roads, where it used to be guided by the light of demonstration, is one of the most singular facts in the history of human knowledge."* It is an equally singular fact, it seems to me, that a writer so generally careful and exact as the author of the "Dissertation on the Mathematical and Physical Sciences," should with so much confidence and amazement, express the above decision as his deliberate opinion; and yet that he should, in the course of a few pages, declare with equal emphasis, that "it is not an accurate account of the controversy to say that it was about words merely."†

The advocates of the above view of the subject contradict themselves, not only by their own express declarations, but also by the conclusions at which they arrive. It was contended by one side, that the force of

* Dissertation, Part II. Sec. 2, p. 54.

† Ibid. p. 59.

a moving body is measured by its velocity simply; and by the other, that this same force is measured by the square of the velocity. Surely, these two measures of one and the same thing cannot be so widely different, and yet both be equally correct. It is not possible for the human mind to maintain so glaring an absurdity. Both propositions cannot be regarded as universally true, and if we suppose that each is true, only within certain limits, or under certain circumstances, then the great error of both sides consisted in setting up a partial truth for a universal maxim in philosophy. Each party, by too hastily deducing a universal proposition from a few particulars, and by struggling to establish this in the face of opposing facts, would be guilty of a great error, and of a great offence against the true method of philosophizing. It certainly cannot be true, then, that both parties, in their ideas or opinions exactly agreed, and yet that at the same time, each endeavored to establish a universal proposition to the entire exclusion of that maintained by the other. Yet, contradictory as these positions are, they are both maintained by D'Alembert, Playfair, and their followers. As if guided by the authority of D'Alembert, Dr. Playfair has pronounced the whole dispute about words merely; and yet, after he had considered the matter for himself, he declared that this was "not an accurate account of the controversy." For he too clearly saw, and has expressly asserted, that the error of each side "*lay in ascribing generality to propositions that are true only in particular cases.*" It surely cannot be necessary to dwell longer on an opinion, which has been so amply refuted by the abettors of it; and, indeed, I should not have deemed it worth while to notice it at all, if it had not found its way into almost every work on mechanical philosophy, which has appeared since the *Dynamique* of D'Alembert.

The solution of the French philosopher does not appear perfectly satisfactory. On the contrary, it seems attended with several insuperable difficulties. In the first place, it presents "two measures of the same thing, the force of a moving body, which are quite different from one another." Now, how can this be? How is it possible that one and the same thing can be measured sometimes by one quantity, and sometimes by a widely different quantity?

Secondly. Let us suppose that two equal bodies, S and D, are projected vertically upward; the one with a single velocity, and the other with a double velocity. Now, it is conceded on all sides, that the force which gives the double velocity to D, is just double of that which imparts the single velocity to S. Yet, because D will ascend four times as high as S, it is concluded that it moves with four times the force. Hence, we are required to believe not only that force is the cause of force, but, also, that a double cause produces a quadruple effect!

Thirdly. This celebrated solution measures the cause by its effect; and, at the same time, denies that the effect is proportional to its cause. But is it not absurd to estimate causes in this way, if they be not proportional to their effects?

Fourthly. It is universally conceded, that if, in the case above supposed, D were met at the outset by a double projecting force, acting downward, in direct opposition to the equal double impulsive force acting upward; then the body would not move at all. It would be simply held *in equilibrio* by the two opposite and equal forces. And yet we are required to believe that it would have moved with a quadruple force, if it had not been checked, and its whole motion destroyed, by a double force!

Fifthly. If one of the effects of a moving body be simply as its velocity, and another as the square of its velocity; it will necessarily follow, that the effect is not always proportional to its cause. And if the maxim that an effect is proportional to its cause be unfounded, then all reasoning from effect to cause is at an end, and the foundation of all knowledge is insecure. We could not even reason from the grandeur and magnificence of the creation to the glory of the great Creator. D'Alembert, who was an atheist, did not fail to perceive this consequence of his doctrine; and has, in express terms, called in question the fundamental law of belief respecting the proportionality of causes and their effects. "Cette diversité d'effets," says he, "provenant tous d'une même cause, peut servir, pour le dire passant, à faire voir le peu de justesse et de précision de l'axiom pretendu, si souvent mis en usage, sur la proportionalité des causes à leurs effets." It is very remarkable, that neither Dr. Playfair nor Mr. Whewell, nor other Christian writers, seem to have been aware of the bearing of the speculations of D'Alembert on the above great maxim of philosophy, as well as on the fundamentals of all religion. On the contrary, they reproduce these speculations as if they were perfectly innocuous and harmless, instead of being, as they are, infected with the virus of the rankest atheism in religion, as well as the most deadly skepticism in philosophy.

The errors above noticed lie comparatively on the surface of D'Alembert's opinion. This opinion is involved in a still deeper error, in a still more radical defect; one which vitiates, not only the commonly received solution of the problem in question, but also other very important portions of the dynamical speculation of the present day. The fallacy here referred to, consists in a virtual denial of the great fundamental law of *in-*

ertia, and the assertion of the inherent power and activity of body.

Galileo is not the only great philosopher, who, since the time of Kepler, has involved his mind in a cloud of sophistical reasoning, by closing his eyes on the light of the law of *inertia*. It is one thing to strike out a great principle in philosophy, and quite another to follow out this principle to all its consequences, so as to remove everything from the whole mass of human knowledge that is inconsistent with it. The first has been frequently accomplished by the efforts of a single genius. The last is usually the laborious work of ages.

The truth of this remark is seen in regard to the law of *inertia*. The glory of no period in the history of a science ever equalled that of a Leibnitz and a Newton. Yet was the transcendent glory of that epoch obscured by false theories, and enigmatical views, erected on an assumption diametrically opposed to the great law of *inertia*. By overlooking this law, that age found itself involved in a controversy from which it could not extricate itself; and from which, as we shall see, the succeeding age escaped only by placing it upon a wrong foundation.

Both sides of the great controversy overlooked the law of *inertia* in their speculations. It was assumed by one party, and conceded by the other, that the space passed over by the moving body is the effect of some force, and is therefore the proper measure of that force. This vague notion of the relation between cause and effect was a source of infinite perplexity and confusion. In reality, the space passed over by a moving body is the effect of no force whatever. The only effect in such a case is the change from a state of rest to a state of motion; and this change once produced, this velocity once generated, the body will continue to move by virtue of the

law of *inertia;* and if there were no retarding force, it would move on in a right line forever. The space passed over arises from the law of *inertia*—the inability of the body to stop itself. The space described is the effect of no cause; or, in other words, is no effect at all. *It were surely absurd, to admit that it requires a cause to prevent a body once set in motion from passing over space, and yet demand a cause to make it pass over space.* Its velocity being generated, the law of *inertia*, or the perfect passivity of a body, and the entire absence of all cause or force, is all that is required to account for the space described.

The space described by a moving body is the measure of no force; unless we take into consideration the time during which it is described. For, if we leave out the consideration of time, the slowest motion may, in consequence of the law of *inertia*, carry the body over the greatest conceivable space. The only thing necessary to such a result is the allowance of sufficient time. But when the *force* of an ascending body, as it is called, is measured by the height to which it rises, the time is not at all considered. For, although a double initial velocity will carry a body four times as high as a single initial velocity, yet will it require twice as long for it to do so; a most important circumstance, and one which is uniformly overlooked by the advocates of the *vis viva*. If we should estimate the relative forces of two moving bodies, by comparing the space one would describe in one second of time, with the space the other would pass over in ten seconds of time; every one would see the absurdity of the proceeding, and cry out against such an utter oblivion of the law of *inertia*. But great as such an error is, it is precisely that which, as we shall most clearly see in the next lecture, lies wrapped up

and concealed in all the reasoning in favor of the *vis viva.*

Even Maclaurin, while treating of this very controversy, in a paper which obtained the prize of the Academy of Sciences at Paris, in 1724, habitually speaks of the space described as the effect of the force to be measured. Hence, although his assault on the reasoning of Leibnitz is to be ranked among the great efforts of the human mind; yet his victory was neither complete nor satisfactory. In point of fact, the logic of Leibnitz, so far from having been exploded by him, succeeded, after a conflict of fifty years, in finding a "local habitation and a name" in the domain of science.

Indeed, the very question proposed, and so strenuously discussed, betrays an inattention to the doctrine of *inertia.* Each party endeavored to find a measure for "the force of a moving body." No one, it seems, during the whole of this protracted controversy, ever stopped to inquire whether there is any such thing as the force of a moving body. No one seems to have asked himself, whether the motion of the moving body may not be something different in its nature from the force by which it is produced. If it should after all, then, turn out that there is no such thing as that of which both parties sought the measure, it will not be at all surprising that their disagreement should have been so obstinate, and their dispute so protracted. For it must be a most difficult task to bring even the greatest philosophers to an agreement, respecting the measure of a mere metaphysical nonentity, which owes its dimensions to the density of the fog through which it is viewed, and its proportions to the plastic power of the imagination.

"One of the effects," says Playfair, "produced by the action of the moving body," is the height to which it

will ascend against the action of gravity. This height, says he, it is well known, is proportioned to the square of the velocity, and hence, if we consider this effect, we are bound to conclude that the force of the moving body is, in this instance, proportional to the square of the velocity. Such is the argument which has been used from the time of Leibnitz down to the present day. Mark the words: *the effect produced by the action of the moving body!* In reality, the motion of the body, and its passage over space, are one and the same thing; and hence, to say that the motion of the body is the cause, and the space passed over is the effect, is to say that a thing is the cause of itself! It is to say, that a body passing over space is the cause of its passing over space. Such is the naked sophism to which the argument of D'Alembert and Playfair is reduced, when stripped of the ambiguities of language, and the illusion of the senses.

In truth, the dynamical philosopher has to deal with only two things, namely, the force by which motion is produced, and the motion which is produced by the force. The one is the cause; and the other is the effect. We have no need of any third thing. We consider the force that produces motion, and the motion produced; but as to "the force of the moving body," as it is called, we know nothing, and we desire to know nothing. We lay it aside as a mere figment of the brain, as a mere dream of the imagination. We shall demonstrate the true theory of motion without its aid. The force which acts upon the body to put it in motion, and the motion of the body, are all the elements for which we have any need. These alone, we believe, will explain all the phenomena, and cause them to appear in a light so clear and simple, that their true nature and significancy cannot be mistaken.

By discarding that metaphysical nonentity, "the force of a moving body;" and relying solely upon the two elements of *force* and *motion*, the one considered as cause and the other as effect, and always refusing to call force *motion*, or motion *force;* I shall hope to produce a demonstration of the true doctrine, so clear and so incontrovertible, that the natural philosopher will be led to reverse his former verdict, and to place the *vis viva* of Leibnitz in the same category with his "living monads," and his "pre-established harmony."

LECTURE III.

THE MEASURE OF FORCE.

THE object of this lecture, as I have already intimated, is to demonstrate the connection and dependency of the phenomena relied on by Leibnitz, with the simple laws of motion as laid down by Newton. Or, in other words, to explode the doctrine of the *vis viva*, and restore the old theory of motion to its original simplicity and beauty.

During the whole of the controversy in question, both sides referred the phenomena of moving bodies, when projected vertically upwards, to the action of one force alone, which they called, as we have seen, "the force of the moving body." This seems most truly wonderful; for it is clearly not the case. Indeed, this vague, confused notion of "the force of a moving body," seems to have misled the power of the mathematics from its true point of application to the question; and thereby rendered a very simple matter exceedingly obscure and difficult. When a body is projected upward against gravity by a certain percussive or impulsive force, this force is one cause. But while the body, set in motion by the projectile force, moves vertically upward, it is continually drawn downward by the force of gravity. This force of gravity is another cause. These two forces are the only causes in the case, and they are amply suffi-

cient to account for the phenomena so confidently relied on by Leibnitz and his followers. Not with "the force of a moving body," nor with any other single force, whether real or imaginary, shall we endeavor to explain the facts on which the commonly received notion of the *vis viva* is founded. We shall simply take the two well-known forces, the force of gravity and the projectile force, and show that the phenomena adduced by Leibnitz to overthrow the doctrine of Newton, are, in reality, the necessary consequences of that doctrine.

To render the subject perfectly clear, and leave no obscurity in the path behind us, let us begin with a particular instance. Let us suppose, that a body is projected perpendicularly upward with an initial velocity of one hundred and fifty Paris feet per second; and that another body, exactly equal to the first, is projected upward with a double force. This, according to the doctrine of Newton, will give it a double velocity; that is, a velocity of three hundred Paris feet per second. The first body will ascend three hundred and seventy-five feet, before it begins to fall; and the second will reach the height of one thousand five hundred feet. This distance is just four times the height ascended to by the first body; that is, $375 \times 4 = 1500$. Now, from this fact, Liebnitz concludes that the force of the second moving body is four times the force of the first. This inference, we say, is badly drawn, and its fallacy may be clearly shown.

If the force which imparted an initial velocity of one hundred and fifty Paris feet to the first body, had acted alone, the body would have passed over one hundred and fifty feet the first second. But the force did not act alone; the velocity it imparted is counteracted by the force of gravity, which continually pulls the body

downward. But the effect produced by gravity on any body at rest is well known; it will cause the body to fall fifteen Paris feet in one second of time. Hence, while the velocity generated by one force would have carried the body up one hundred and fifty feet, that generated by the other force draws it down fifteen feet; and, consequently, instead of ascending one hundred and fifty feet the first second, the body actually ascends 150 — 15, or one hundred and thirty-five feet. In two seconds, gravity causes a body to fall sixty feet; this taken from three hundred, the height to which the body in question would have ascended if gravity had not acted, leaves 300 — 60, or two hundred and forty, for the actual height reached by the body under the joint influence of the two forces. By continuing the same process, we find that the body will, at the end of the third second, be 450 — 135, or three hundred and fifteen feet high; at the end of the fourth, 600 — 240, or three hundred and sixty feet high; and at the end of the fifth, 750 — 375, or three hundred and seventy-five feet high. Here, all its motion upward will be exhausted, and it will begin to descend.

Now, let us mark the result. The body was projected upward against gravity with an initial velocity of one hundred and fifty feet per second. If gravity had not acted, this velocity would have carried the body over the distance of seven hundred and fifty feet, in five seconds. But gravity continued to act, and constantly retarded the velocity generated by the projectile force. Hence, instead of ascending seven hundred and fifty feet, the body only reaches the height of three hundred and seventy-five feet, and then begins to descend. But a body having a uniform velocity of seventy-five feet per second, will pass over three hundred and seventy-five feet in five seconds. The actual motion of the body, then, is exactly

equivalent to a uniform velocity of seventy-five feet per second for five seconds.

Let us now consider the second body. This is projected upward with a double force, and consequently receives a double initial velocity; that is, a velocity of three hundred feet per second. By repeating the process applied to the first case, we may easily ascertain, that the body now under consideration will continue to ascend for ten seconds only, and will reach the height of one thousand five hundred feet before it begins to fall. This height is equal to the space described by a uniform motion of one hundred and fifty feet per second continued for the same time, or ten seconds. All the actually upward motion in this case, then, is exactly equivalent to a uniform motion of one hundred and fifty feet per second continued for ten seconds; that is, for the time during which the body continues to ascend.

It is easy to compare these results. In the first case, the body is projected upward by a certain force, which imparts to it an initial velocity of one hundred and fifty feet per second. It ascends for five seconds, and reaches the height of three hundred and seventy-five feet. This is equivalent to a *uniform motion*, as we have seen, of seventy-five feet per second, continued for five seconds. In the second case, an equal body is projected upward by a double force, and consequently, according to the doctrine of Newton, receives a double velocity; that is, a velocity of three hundred feet per second. It ascends for ten seconds, and reaches the height of one thousand five hundred feet. This is equal to the space passed over by a uniform motion of one hundred and fifty feet per second, continued for the same time. Hence the mean velocity in the second case, is double the mean velocity in the first case; it continues for double the time, and it describes four times the space. Now, is it

very wonderful, that a double velocity continued for a double time, should describe a quadruple space? It is certainly the natural and necessary result of the very law laid down by Newton, and it is in perfect accordance with the equation of uniform motion which has been universally adopted, from his time down to the present day. This great seeming exception to his doctrine, then, is in reality a strong confirmation of it.

In like manner, it may be clearly shown, that a body projected vertically upward against gravity by a triple force, and with a triple velocity, will ascend to nine times the height; because it will proceed with three times the velocity for three times the number of seconds; and if projected upward with four times the velocity, it will ascend to sixteen times the height; because it will proceed with four times the velocity, for four times the number of seconds. Hence the facts adduced by Leibnitz to overthrow the doctrine of Newton and Descartes, are the necessary results of that doctrine; and that doctrine furnishes the only rational and satisfactory explanation of them. They do not prove that the *force* is proportional to "the square of the velocity," they only prove the old doctrine, that the space is proportional to the velocity into the time. That is to say, the grand enigma of Leibnitz, when closely analyzed and searched to the bottom, simply amounts to this, that a double velocity, continued for a double time, will describe four times the space! Is it not wonderful, that this enigma should have entrapped such minds as those of Leibnitz, Bernouilli, D'Alembert and Playfair; or that it should so long have resisted the assaults of such adversaries as Maclaurin, Clarke, Jurin, and Mairan?

Let us turn this matter in another light. Suppose the action of gravity to be suspended; and if we must speak of the "force of a moving body," let us consider

this force as it acts alone. One body, say, moves with a velocity of one hundred and fifty Paris feet per second; and the other with a velocity of three hundred feet. Now, if we would compare these two forces, we must consider the spaces they would describe, *in the same time.* In five seconds, the one will pass over, seven hundred and fifty, and the other one thousand five hundred, feet. Hence, the one is just double of the other, according to the only fair way of estimating them. But if we suppose the last to move ten seconds, with its velocity of three hundred feet per second; it is clear that it will pass over three thousand feet, or four times the space described by the first in five seconds. Now, would it not be ridiculously absurd to compare these spaces—the space described by the single velocity in five seconds, and the space described by the double velocity in ten seconds; in order to estimate the ratio of the two forces? Most assuredly it would. And yet this absurdity, glaring as it is when laid open, is precisely that committed by D'Alembert and Playfair.

This subject may be viewed in still another light. The force of gravity, you know, is considered as a constant force, in all cases of falling bodies near the surface of the earth; and it was so regarded in the controversy under consideration. To view this question still more closely, then, let us suppose a centre of attraction, which exerts an equal force at every point around it. Let us also suppose, that this equal, constant force will cause a body, at any distance from it, to fall toward it 15 Paris feet in one second of time. A body, then, beginning to fall just 15 feet from the centre of attraction, would reach it in exactly one second. Another, starting from rest at the distance of 60 feet, would reach it in exactly two seconds. For the attractive force would make it describe 15 feet in one second; and then, if this force

were suspended, it would pass over 30 feet the next second, by virtue of the law of *inertia* alone. But the attractive force acts during this last second also; and makes the body describe 15 feet more; and the whole space passed over in two seconds is 15+30+15, or 60 feet. Now, *ex hypothesi*, the force exerted in the one case is just double of that exerted in the other; while the one space is quadruple of the other. Hence, it is clear that the spaces described by the two falling bodies cannot be taken as the measures of the respective forces, and that, if we do so, we overlook the law of *inertia*, and sink it in oblivion.

But, after each of the two bodies has reached the centre in question, it will begin to ascend, or, in other words, it will begin to pass away from the centre. The one, or that which has fallen over 15 feet, and acquired a single velocity, will pass 15 feet beyond the centre before it begins to return, or fall again. The other will rise, or pass beyond the centre, to the distance of 60 feet. Shall we adopt these spaces, then, as the measures of the forces of the two moving bodies? If so, we shall be compelled to pronounce the one four times as great as the other; but this would be manifestly inconsistent with the hypothesis. For, by the hypothesis, one has been generated by a constant force acting for one second, the other by the same force acting for two seconds; and consequently the last is just double of the first. Again, the one has been extinguished by a constant force acting for one second, the other by the same force acting for two seconds; and therefore the one is just half of the other.

In this connection, I shall call your attention to a very remarkable passage in the work of Dr. Playfair. "When a moving body is opposed by pressure, by a *vis mortua*, or a resistance like that of gravity, the quantity

of such resistance required to extinguish the motion, and reduce the body to rest, must serve to measure the force of that body. It is a force which, by repeated impulses has annihilated another, and these impulses, when properly collected into one sum, must evidently be equal to the force which they have extinguished. *It happens, however, that there are two ways of computing the amount of these retarding forces, which lead to different results, both of them just, and neither of them to be assumed to the exclusion of the other.*" What! two ways of computing the amount of the same quantities, which lead to different results! Surely, if arithmetic be a true science, both ways cannot be right. The author tells us that both ways are just, and neither of them should exclude the other; but if so, we must, even in mathematics, walk by faith, and not by sight or reason. But let us see these two ways. "Suppose a body, the force of which is to be measured, to be projected perpendicularly upward with any velocity, then, if we would compute the quantity of the force of gravity which is employed in reducing it to rest, we may either inquire into the retardation which that force produces during a given time, or while the body is moving over a given space. In other words, we may either inquire how long the motion will continue, or how far it will carry the body before it be entirely exhausted. If the length of the time that the uniform resistance must act before it reduce the body to rest be taken for the effect, and consequently for the measure of the force of the body, that force must be proportional to the velocity; for to this the time is confessedly proportional. If, on the other hand, the length of the line which the body describes, while subjected to this uniform resistance, be taken for the effect, and the measure of the force, the force must be as the square of the velocity, because to that quantity

the line in question is known to be proportional." . . . "Here, therefore, are two results, or two values of the same thing, the force of a moving body, which are quite different from one another."

Now, the one of these ways, I contend, is true, and the other false. We should measure "the force of the moving body," as it is called, by the time it will take gravity to extinguish its motion upward. For, it is well known, that the force exerted by gravity is proportional to the time during which it acts. Hence, as the initial velocity of three hundred feet in the case supposed is overcome, or neutralized, by gravity in ten seconds, while half that velocity is overcome in half that time, it follows that it moves with the double of the force of the body setting out with half its velocity. The motion of the moving body, or the force of the moving body, (call it what you please,) which is generated by a double force, and destroyed by a double force, must itself be double. Hence, the force of three hundred feet per second, must be double of one hundred and fifty feet per second; for it is confessedly generated by a double force, and it is extinguished in a double time by the action of gravity. And surely, if anything be plain, gravity exerts twice the force in ten seconds that it does in five.

This is the only true way of estimating the amount of the impulses exerted by gravity. The other way is false. Gravity exerts the same force on a body, whether that body move slower or faster; and hence, the space passed over by the body, while it is being exerted, has no fixed relation to its amount, and cannot serve as its measure. Whether a body move one foot, or ten, per second, the force of gravity exerted on it is the same; and, therefore, the space passed by the body while gravity acts upon it, has nothing to do with the amount

of gravity exerted. Such a measure of the force of gravity is most unsound, and, if carried out, would lead to the wildest and most absurd conclusions.

But, let us suppose, for the sake of argument, that the impulses of gravity are properly summed up, and measured by means of the space which the ascending body describes, while they are exerted. It will then follow, that gravity exerts four times the force in two seconds that it does in one, or in ten seconds that it does in five. For the body which ascends fifteen hundred feet, is overcome by gravity in ten seconds, while that which rises three hundred and seventy-five, begins to return in five seconds. If these spaces are to be adopted as the measure of the force of gravity exerted during those times; then gravity exerts four times as much force in ten seconds as it does in five. But this is utterly inconsistent with the fact, that gravity is a constant force, and always exerts the same influence in the same time.

We may show that the sum of the impulses, by which the motion of a body is destroyed, cannot be measured by the space which it describes while the impulses are exerted, by a very simple and conclusive illustration. If a body be acted on by diametrically opposite forces at the same time, it is well known it will not move at all. For example, if each of the two forces would produce a velocity of one hundred and fifty feet per second, the body would remain at rest. Now, let us suppose that the whole of one of these opposite forces is made to act on the body at once, and only half of the other equal force. The body will then begin to move with a velocity of seventy-five feet per second; and if, at the end of one second, the other half of the force act in direct opposition to its motion, the body will be instantly reduced to a state of rest, and it will pass over only

seventy-five feet in all. But, suppose again that this force, instead of being divided into two, is divided into five equal parts, or impulses; and that these are successively applied to the body at the beginning of the first, second, third, fourth, and fifth seconds of its motion. In this case, the sum of the impulses is precisely the same as in the last instance; and yet, the body, instead of passing over only seventy-five feet, actually describes three hundred feet. The reason of this difference is, that the law of *inertia*, which is overlooked by the advocates of the *vis viva*, has had a longer time to operate in the one case than in the other. The sum of the counteracting impulses is the same in both cases, but as this sum is divided into a greater number of equal parts, and spread over a greater number of equal intervals of time, in the one case, than in the other; so the body in one case is allowed by virtue of the law of inertia to pass over a greater space. And, supposing the intervals of time, at which the equal impulses are applied, to be equal, the space described will be longer or shorter, as the number of impulses into which the force may be divided is greater or less. All this is perfectly applicable to the effect of gravity, which may be conceived to be divided into infinitely small impulses, and each one applied at equal intervals of time. Hence, the space cannot be the measure of the sum of these impulses; for the very same sum admits, as we have seen, of a greater or less space, as its equal portions are spread over a greater or less period of time. For the same reason, if the sums were not equal, but one was just double of the other; the one that is spread over the greatest time will allow the body to describe a disproportionately greater space. Hence it is, that although the sum of those impulses of gravity, by which the velocity of three hundred feet is destroyed, is just double

of that by which the velocity of one hundred and fifty feet is extinguished; yet the first describes a quadruple space while the double sum is being exhausted. If the two sums acted upon their respective bodies at once, or if both were exhausted in the same time, they would bring both bodies to a state of rest at the same time; but the equal portions of the double sum are distributed over a double time; and therefore, *in consequence of the law of inertia*, the double velocity describes a quadruple space.

If necessary, we might, by various other illustrations, show the fallacy involved in the *vis viva*. But I forbear, lest the discussion should become insufferably prolix and tiresome. In conclusion, we shall notice a few of the blunders in practice which have sprung from the foregoing error in theory.

Though Descartes adopted the mass into the velocity as the measure of the "force of a moving body;" yet, when that force is made to act in direct opposition to gravity, he measured it by the mass into the height to which the body ascends. He did not perceive the incompatibility of these two measures. Hence, the Cartesians gave Leibnitz and his followers a great advantage over them. In his first paper, Leibnitz availed himself of the unsound portion of the Cartesian theory, with irresistible effect. It is conceded by the Cartesians, said he, that the same force which projects a four-ounce weight one foot high against gravity, will throw a one-ounce weight four feet in the same direction. And this was conceded by them, though the admission is at war with the true portion of their theory.

This may be easily shown. If, with Descartes, we measure the projectile force by the mass into the velocity; then the same force will impart four times as great a velocity to one ounce as it will to four ounces.

But this quadruple velocity, if in a vertical direction, will carry the one body sixteen times as high as the single velocity will carry the other. Such is the true consequence of the Newtonian and Cartesian doctrine; and it is a great pity, that M. L'Abbe De Conté, and others, who undertook to defend that doctrine, did not see this to be the case. Instead of admitting, as they did, that the force which projects four ounces to the height of one foot, will throw one ounce four feet high; they should have concluded, that it would, as the legitimate consequence of their theory, cause it to ascend sixteen feet.

The common error of measuring the force by the mass into the height, which so long darkened the subject in question, and confounded the disputants on both sides, still holds a place in the science of dynamics. M. Boucharlat, for example, says: "In this second method of considering the effects of forces, we regard them as proportional to the weight raised, and the height to which it is elevated in a given time; *it being always understood that the effort necessary to overcome the weight, is not supposed to vary with the elevation.* If, for example, two men raise the same weight, in the same time, to the heights of six hundred and two hundred yards respectively, we would, according to this method of estimating the effects of forces, regard the first as possessed of three times the force of the second. Again, if, in the working day, one man can raise a weight of fifty pounds through a height of two hundred yards, and a second a weight of twenty-five pounds through a height of four hundred yards, we should regard the two men, according to the present hypothesis, as possessed of equal strength, although the absolute strengths of the two might be very different; the strengths of the two individuals are here considered only with reference to

the work done. This method of estimating forces was adopted by Descartes."

From this measure, which is, as we have seen, the unsound part of the Cartesian doctrine; it was easy for M. Boucharlat to demonstrate, as he has done, that the force is equal to the mass into the square of the velocity.

To try the justness of this measure as a practical rule, let us suppose that the strength of one man is just equal to five hundred pounds, while that of another is a little greater. Now, if the first lifts at the five hundred pounds, he can by the hypothesis, relieve the earth of its pressure, but he cannot raise it from the surface of the ground. According to the measure in question, then, his strength is equal to 500×0, or *nothing*. But as the other can raise the weight, say an inch, his strength will, according to the same measure, be 500×1, or 500. We should conclude, then, from this method of estimating the two forces, that the one is to the other as 500 is to nothing; whereas, in reality, they are very nearly equal.

In like manner, if one machine could just raise 4,999 pounds, and another 5,000 pounds, we might, by applying the above method of estimating their forces, prove that one is infinitely more powerful than the other. These are "glaring instances;" but the same error, to a greater or less extent, runs through all the measures of forces which are made according to the method in question. The truth is, that, in such cases, the natural philosopher aims to measure, not the force exerted by the man or the machine, but the difference between this force and the force of gravity. Is it not wonderful, that a consideration so very obvious should so long have escaped the attention of the mathematician?

Precisely the same oversight is committed in the

well-known experiment of the whirling table; the object of which is to demonstrate to the eye, that the centrifugal force generated by a circular motion, is as the square of the velocity. It is attempted to show, by this experiment, that a double circular velocity will cause four ounces to ascend in the same time that a single circular velocity will raise one ounce.

But in this experiment, the effect is produced, not by the centrifugal force alone, but by the difference between this force pulling the body upward, and the force of gravity drawing it downward. And the experiment will work well, the illusion of the ocular demonstration will be perfect, not when one centrifugal force is quadruple the other, but when the excess of one over gravity is four times the excess of the other. This accounts for the repeated failures of the experiment in question; which the experimenter usually ascribes to some disorder in his machine. The disorder is in his method, in his thoughts; and, with the most perfect whirling table ever constructed, it were easy to raise a six, or an eight, or a ten-ounce weight, by means of the double circular velocity, in the same time that the one-ounce weight is raised by a single velocity.

In conclusion, young gentlemen, if I have satisfied you that the force of a moving body should never be measured by the square of its velocity, but always by the velocity simply, our time has not been misemployed. For if we are right on this subject, then have we exposed errors which most deeply affect the very vitals of mechanical philosophy, and which, for more than a century and a half, have embroiled the clearest and most important principles of the science. Then have we demolished the *vis viva* of Leibnitz, which has so long disfigured every system of rational mechanics, as well as taxed the brain and exhausted the ingenuity of its

cultivators. And we may now, as the result of our labors, repudiate two measures of one and the same thing, widely different from each other, as an impossible and contradictious conceit—as a flagrant violation of the unity, harmony, perfection, and beauty, of scientific truth. We may look through appearances, and behold the very nature of things themselves. We may behold, with undisturbed vision, and with increased delight, that simple but sublime theory of motion, which forms the golden secret of the material universe; beneath whose touch every cloud is dispelled, every enigma solved, every phenomenon explained; and the Divine order of the universal frame, of the whole magnificent panorama of visible things, is laid open and rendered apparent to the human eye.